

Shock Propagation And Attenuation In Bubbly Liquids

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LONG-TERM GOAL

Our long-term goal is to provide a simple yet accurate mathematical model for the generation, propagation and attenuation of acoustic shock waves in bubbly liquids. This model is to be based upon a continuum treatment of the bubbly liquid, in which the dynamics of the many bubbles that are present in the liquid need not be followed individually. Rather, only their overall effects on the properties of the continuum are to be taken into account in the model. Our study is motivated by trying to understand shallow water acoustics and the acoustics of the region near the ocean surface, where wave breaking entrains air bubbles in the water.

OBJECTIVES

Existing experiments on shock propagation in bubbly liquids have exhibited a rich variety of phenomena, many of which are not well explained by current mathematical models. For instance, depending on the state of the system, in the region immediately behind the shock, oscillations in pressure and density of the medium have been observed. The character of such oscillatory shocks depends strongly on experimental conditions. The oscillations can occur about the higher value (in density or pressure) which is achieved behind the shock, or they may occur about an intermediate level between the low and high values, followed by a slow monotonic transition to the higher state. Our scientific objective is to construct a simple continuum model, which includes the requisite physics to be able to provide qualitative and quantitative descriptions of the experiments and to be used as a predictive tool for modeling nonlinear acoustic phenomena in bubbly liquids.

APPROACH

Our approach has been based upon the use of a recently proposed equation-of-state (EOS) for bubbly liquids, obtained in our research group. It suggests that the medium can be treated as a standard continuum, but with pressure in the medium related to the density and its first two substantial time derivatives. This functional form is obtained upon recognizing that the density of the bubbly liquid has a one-to-one relationship with the radii of the bubbles in the mixture. Since the bubble radii satisfy the Rayleigh-Plesset equation which is a second-order ODE relating the radius and its first two time derivatives to the mean pressure in the vicinity of the bubble, this gives rise to the above functional form for the equation of state. Upon combining the above EOS with the equations of continuity and Euler for the continuum, a very straightforward model results for describing nonlinear wave phenomena in bubbly liquids. The validity of such an EOS relies on the assumptions that the bubbly liquid is monodisperse and that the bubble phase does not move relative to the liquid phase. Relaxing these

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assumptions complicates the EOS, but in a manner that may still be tractable. For instance, if allowance for relative motion between the two phases is made, the pressure is found to depend on both the mixture density and the number density of bubbles, as well as both of their first and second time derivatives. This last generalization has, in fact, been one of the major accomplishments in our research group within the past year.

WORK COMPLETED

As a first step in assessing the capabilities of this approach, as reported in our previous progress report, we had considered a one-dimensional propagation problem, modeled by the full nonlinear continuity and Euler equations, together with a *linearized* approximation to the above equation-of-state. A traveling-wave solution of the resulting system of equations resulted in a simple second order nonlinear ordinary differential equation (ODE) that was readily analyzed using its phase portrait. Since then, we have been able to extend the analysis in two very important respects. First, it was found to be possible to make use of the *fully nonlinear* equation-of-state, resulting in a description that not only matches the qualitative features of shocks in bubbly liquids, but also provides good quantitative agreement with experiments. Furthermore, we have been able to complete a detailed analysis in the case where relative motion between the bubble phase and the rest of the mixture is allowed.

RESULTS

For the case where relative motion (slip) between the bubble phase and the liquid is allowed, a complete set of continuum level governing equations has finally been identified. These consist of the mixture conservation of mass and momentum equations, which have their standard forms. The pressure in these equations, however, is found to depend on the mixture density and on the number density of bubbles, as well as their first two substantial time derivatives, taken with respect to the velocity of the bubble phase. We thus need an additional relationship between the velocity of the bubble phase and that of the mixture. This turns out to be a bubble-phase momentum equation in which account needs to be taken of the added-mass effects, the "acceleration reaction," and the viscous drag on the bubble phase by the surrounding liquid. In the traveling wave approximation, these equations can be reduced to a set of four nonlinear first order ODEs whose dynamics can be a lot richer than the two degree-of-freedom system that results when slip is not accounted for. In particular, the shock waves can now oscillate about an intermediate value of density (or pressure) between the front and back of the shock, followed by a slow transition to the higher value behind the shock, in agreement with some experiments. We find that even for very small bubbles (e.g. 10-100 microns) the relative motion, or slip, between the bubble phase and the surrounding liquid turns out to be quite important.

IMPACT/APPLICATIONS

To be able to treat the bubbly liquid as a continuum, without the need to calculate the detailed dynamics of the individual bubbles that exist in the medium, represents a very powerful advance in our modeling capabilities. With the aid of our non-equilibrium equation-of-state, relating pressure to mass and number densities and their first two time-derivatives, more efficient computational schemes can be devised to serve as predictive tools for modeling the generation, propagation and attenuation of nonlinear waves in such media. We are in the process of devising high-precision numerical methods that are capable of handling nonlinear acoustic phenomena in such bubbly mixtures.

TRANSITIONS

Our research is continuing along several fronts. The full nonlinear EOS, also accounting for slip between the bubble phase and the liquid, is being implemented in the traveling wave solution and the differences between its predictions and those of the linearized EOS are being assessed. At the same time, to follow the transient dynamics, the full equations of continuity and Euler and the nonlinear EOS are being integrated numerically using well-known schemes for systems of conservation equations in one, two and three dimensions, together with novel methods for dealing with the present non-equilibrium EOS.

RELATED PROJECTS

The PI is collaborating with a colleague, Tasso Kaper, of the Department of Mathematics at Boston University, in a project which aims to understand the dynamics of individual bubbles, or small clusters of two, three or more bubbles, using modern methods for treating nonlinear dynamical systems. Together with graduate student Anthony Harkin, the question of determining the dynamic threshold for acoustic cavitation has been resolved, and issues of energy transfer between breathing and shape modes in a single bubble, and interactions among neighboring bubbles are being considered.

PUBLICATIONS

1. J. J. Cartmell, A. Nadim and P. E. Barbone, "Numerical Simulation of Shock Propagation in Dilute Monodisperse Bubbly Liquids," American Physical Society, Division of Fluid Dynamics, Annual Meeting, San Francisco, CA (November 1997). [*Bull. Am. Phys. Soc.*, **42**, 2180 (1997).]
2. Nadim, D. Goldman, J. J. Cartmell and P. E. Barbone, "A Phase-Plane Description of Nonlinear Traveling Waves in Bubbly Liquids," *Journal of Computational Acoustics*, (Accepted, 1998).